Stochastic resonant memory storage device

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We show that an extended system operating in the regime of stochastic resonance can act as a short-term memory device. The system under study is a ring of overdamped bistable oscillators coupled directionally, being each also subject to an external source of Gaussian white noise (the noise sources are independent). A single oscillator is driven by an external periodic force, assumed to act only over the time that the signal takes to traverse the whole ring. A traveling wave is then found to be transmitted several times along the ring with a small damping, provided that the driven oscillator operates in a regime close to stochastic resonance. If noise is suppressed from any oscillator of the chain, the traveling wave is immediately damped. The ring is thus found to act as a short-term memory device in which the stored information (one bit, corresponding to the presence or absence of the external driving) is sustained by noise during a characteristic time T_{mem}.

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I. INTRODUCTION

Stochastic resonance (SR) is by now a well-established phenomenon, consisting of a noise-enhanced coherence of the output of some nonlinear systems with their input. In noise-driven, periodically modulated bistable systems, it manifests itself in the enhancement of periodic transitions at the frequency of the external driving. The external noise and the periodic driving mechanism interact to produce a well-defined spike in the power spectrum of the system. The corresponding signal-to-noise ratio (SNR) has a maximum when the intensity of the external noise is properly tuned to the internal parameters of the bistable system. For larger noise amplitudes the signal is increasingly corrupted. A description of the theory of SR and a comprehensive review can be found in Refs. [1] and [2].

The relevance of this phenomenon to the biophysics of neural excitation has also been investigated in great detail [3,4]. The neuron is modeled as a bistable system in which the two stable configurations correspond to the "quiescent" and "firing" states of the axon [5,6]. The relevance for neural *transmission* has triggered investigations about the occurrence of SR in extended systems. In Ref. [7], SR is studied in a one-dimensional array of coupled oscillators. A collective enhancement is found, associated to a massive synchronization of the whole array, whenever the noise intensity is properly tuned to the coupling between neighboring oscillators.

The relevance of SR to the transmission of signals between neurons has been investigated through the occurrence of a similar process along a chain of bistable devices. The links represent the neurons, the mutual synaptic connections are modeled by the directional coupling of devices along the chain, and the external Gaussian noise models the random (thermal) excitations of the environment to which neurons respond. The coupling is not of a mechanical nature but rather of an electrical type and there is not any delay assumed in the coupling between adjacent oscillators. This model has been studied in Refs. [8–10] and [13].

The problem of noise-sustained transmission is also related to spatiotemporal SR as reported in Ref. [11] (see also Ref. [12]). This situation is found when a spatially extended wavefront impinges on a subexcitable medium that is stimulated by spatially distributed and uncorrelated noise with a Gaussian distributed amplitude. If the dispersion of the noise is properly tuned, the wave front is found to propagate with little distortion at much longer distances and with less attenuation than in the absence of noise. The model that we consider can be regarded as a toy model for this phenomenon because it also involves the noise enhanced propagation of a signal in a spatially extended system and the possible occurrence of a sustained traveling wave in it.

In the present paper we address the possibility that reverberant excitations could be found in extended systems as a consequence of the same transmission mechanism that operates in an open chain. The intuitive picture is the following. Assume that the first oscillator of the chain is driven by a weak, periodic external force and that also Gaussian noise is fed into it. If the oscillator is in the regime of SR, it will perform enhanced periodic transitions. The next oscillator in the chain is then driven by this signal and it is likely to also perform similar large-amplitude oscillations. It has been proven [13] that there is a regime in which noise is essential to propagate the signal from one link to the next. If the chain closes onto itself a traveling wave could then be formed, thus providing the basis for reverberant excitations to be sustained by noise long after the external driving is turned off.

II. THE SYSTEM

We consider a "transmission line" built from N bistable oscillators, such that each node feeds the next one with a

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signal that is proportional to its oscillation amplitude. By shaping the line as a ring, we let the signal of the Nth oscillator be fed back into the first one. The time evolution of the system is thus described by the set of coupled equations

$$\dot{x}_n+\frac{\partial V}{\partial x_n}=F_n^{dr}(t)+\sqrt{D}G_n(t), \quad n=1\ldots N, \tag{1}$$

where x_n is the amplitude of the nth oscillator, and $V(x_n)$ is the corresponding double well potential with minima of depth U_o located at $x_n=\pm c$:

$$V(x_n)=-U_o\left(\frac{x_n}{c}\right)^2\left[2-\left(\frac{x_n}{c}\right)^2\right]. \tag{2}$$

The terms $\sqrt{D}G_n(t)$ represent uncorrelated sources of Gaussian noise of amplitude (variance) D. Each noise has unit power-spectral density.

The driving force F_n^{dr} in Eq. (1) is taken to be proportional to the amplitude of the preceding oscillator in the ring, except for one that we label with $n=1$. That oscillator is instead assumed to be driven by a periodic external force. We also assume that there is a fixed delay τ between one oscillator and the next one, so that the signal spends $N\tau$ units of time to jump across the N links of the ring. By the time the signal reaches again the first oscillator, the external driving force is turned off and it is replaced by the driving of the Nth (last) oscillator, thus closing the chain of mutual excitations. We thus write

$$F_n^{dr}=\begin{cases}\epsilon\cos(\omega_{ex}t) & \text{for } n=1 \text{ and } 0<t<N\tau,\\ \epsilon' x_N(t-\tau)/c & \text{for } n=1 \text{ and } t>N\tau,\\ \epsilon' x_{n-1}(t-\tau)/c & \text{for } n>1 \text{ and } t>(n-1)\tau,\end{cases} \tag{3}$$

where the index n is assumed to run cyclically between 1 and N. In writing the above equations we have assumed that all the oscillators are identical and taken units such that the damping coefficient is 1. In addition, the amplitude of the oscillations is measured in units of the ''width'' c of the biquadratic potential.

The physical parameters of the system are the noise amplitude D, the potential depth U_o, the potential well separation c, the damping coefficient, the couplings ϵ and ϵ', the time delay τ, and the forcing frequency ω_{ex}. We may usefully express D in units of $D_{SR}\sim U_o/2$, which is the ''resonant'' value of the noise at which the first oscillator is expected to have the largest SNR (see Ref. [1]).

The parameters ϵ' and τ specify the transmission. The latter also fixes the input of energy coming from the external driving because power is fed into the system during an interval $N\tau$. We should therefore choose τ such that $N\tau \gg 2\pi/\omega_{ex}$. The parameters ϵ and ϵ' specify the coupling of the oscillators to their respective driving signals and are largely independent from each other. One should, however, expect that $\epsilon'>\epsilon$ because of energy considerations: power fed into the large amplitude oscillations must chiefly come from the external noise. In the coupling with the external periodic signal, power is concentrated in the single frequency ω_{ex}. The coupling between oscillators involves instead a noisy signal and therefore its power is spread over the whole spectrum.

This setup allows a self-sustained traveling wave to propagate along the ring. In this process, noise plays two contradictory rôles. On the one hand it causes an enhancement of the oscillations in each link which makes possible the transmission but on the other, it also progressively corrupts the signal.

The system under consideration can be regarded as one in which several time scales are involved. The shortest one is the (infinitesimal) interval between individual stochastic stimuli. The next one is given by the period $T_{ex}=2\pi/\omega_{ex}$ of the external driving force. This is large compared with the previous one but short compared to τ, that is the delay involved in the transmission from one oscillator to the next. We consider τ to be of the same order of magnitude as $N\tau$, which is the time required by the signal to travel the ring of N oscillators. This assumption restricts the analysis to rings of *few* oscillators. The opposite case can be analyzed as an open chain that has been considered previously in the literature. In this latter case also τ loses any physical relevance.

The model contains several other constants with the dimensions of time. These are c^2/U_o, c^2/D, c/ϵ, and c/ϵ'. The first is related to the Kramers time (see Refs. [15] and [1]) that represents the mean first-passage time or the typical time required to escape from one of the two wells of the potential. This Kramers estimate is made under the assumption that the probability density in each of the valleys of the potential is approximately in equilibrium and given by a Gaussian centered at $x=\pm c$. In order to satisfy this equilibrium condition, the signal frequency must be sufficiently slow, i.e., it must fulfill $T_{ex}\gg c^2/U_o$. The second time constant is of the same order of magnitude as the first one because the system is assumed to operate in the regime of stochastic resonance, which links the values of D and U_o. The time constant c/ϵ can be physically interpreted as the typical time required to travel a distance equal to the width of the well under the influence of the external driving. This time scale is taken to be of the same order of magnitude as T_{ex}. Clearly c/ϵ' plays a similar role as c/ϵ, if the external driving is replaced by the signal of the preceding oscillator in the chain.

As long as the signal is not appreciably damped over intervals shorter than $N\tau$, one can define an even longer time scale by the typical attenuation time T_{mem}. In this case, the ring can be thought to act as a memory device that stores one bit of information—corresponding to the presence or absence of the external driving—for a time of the order of T_{mem}. It is obvious that this typical decay time, if expressed in units of τ has also the meaning of a decay length as the one discussed in Ref. [13].

III. RESULTS AND DISCUSSION

In order to obtain an empirical estimate of T_{mem} we have integrated Eqs. (1). We have chosen the integration step dt to equal $T_{ex}/1024$. We have also chosen $U_o=256$, $c=4\sqrt{2}$, $\omega_{ex}=0.39\pi$ and $\epsilon=8$. We have performed several numerical

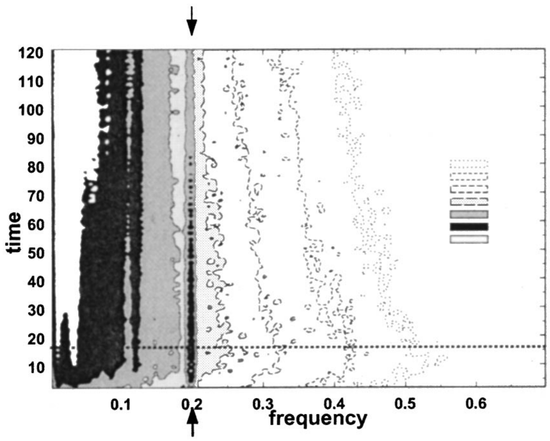

FIG. 1. Contour plot of the time-dependent power-spectral density obtained with the windowed Fourier transform in a ring with $N=16$ links, during eight complete loops. Time is expressed in units of τ, which equals $8T_{ex}$ (i.e., 8192 integration time steps). The lowest values of the power-spectral density are shown only through the contour lines, whereas higher values are shaded with darker levels of gray. The highest value is represented as white. The ridge associated to the traveling wave is located at the same frequency as the external driving and is shown with arrows. The dotted line indicates the instant at which the external driving is turned off. The noise amplitude is $D=0.75D_{SR}$ and $\epsilon'=80$.

simulations involving rings with different numbers of links and delay times τ. These are quoted in the figure captions below. We leave D and ϵ' as free parameters. Since these two govern the transmission properties of an open chain of oscillators [13], they are also expected to largely determine the value of T_{mem}.

We have evaluated the power-spectral density of the first oscillator during L loops of the traveling signal, averaging it over a suitable ensemble of initial conditions ($\sim$100) in order to hinder fluctuations. In order to have a picture of the decaying process we perform a windowed Fourier transform [14]:

$$X_n(f,s)=\int_{-\infty}^{+\infty} H_{N\tau}(t-s)x_n(t)e^{i2\pi ft}dt. \tag{4}$$

The window function in Eq. (4) is $H_{N\tau}(x)=1$ for $|x|\leqslant N\tau$ and 0 otherwise. The resulting transform is a function of the frequency and of the time s where the window is centered. The corresponding time-dependent power-spectral density describes changes in a signal that has a duration equal to the width of the window of $H_{N\tau}(x)$. In Eq. (4) such width is taken to be $N\tau$, consistently with the assumption that the signal is not appreciably damped within such interval.

A contour plot of the time-dependent power-spectral density is shown in Fig. 1. The occurrence of a traveling wave with enhanced transitions as in SR can be seen as a ridge located at a frequency equal to the external driving and parallel to the time axis. The ridge is significantly longer than $N\tau$, thus showing that there is a remnant traveling wave even after the moment at which the external driving is turned off.

The noise corruption can be appreciated not only by the fact that this ridge becomes less prominent but also because the height of the landscape increases at lower frequencies and larger times (the extreme northwest part of the landscape).

The relevance of noise in *sustaining* the traveling wave can easily be checked by depriving of noise one of the links of the ring. As for an open chain [13], we find a critical value $\epsilon'_{crit}\sim 65$ below which the traveling wave is destroyed; but for a larger value, transmission is restored.

Below such critical value transmission is dominated by noise, while above it is increasingly due to the coupling between oscillators.

In order to have a quantitative picture of the storage and attenuation processes, we have analyzed the behavior of the signal-to-noise ratio as a function of the number of loops L traveled by the signal. We consider the following two relative attenuation coefficients (where the SNR is denoted as R):

$$A_{in}=\frac{[R(1)-R(2)]_{dB}}{[R(1)]_{dB}}, \tag{5}$$

$$A_{st}=\frac{[R(2)-R(2+\Delta L)]_{dB}}{\Delta L[R(2)]_{dB}}, \tag{6}$$

In Eqs. (5) and (6), $R(L)$ stands for the signal-to-noise ratio that corresponds to the Lth loop. The notation $[\cdots]_{dB}$ indicates that the quantities between brackets are measured in dB.

The coefficient A_{in} is the relative loss in SNR from the first to the second loop, i.e., it compares the SNR *while* the system is being driven with that *immediately after* the information has been stored. This coefficient gauges the attenuation involved in the input process.

The quality of the storage is related to A_{st}. This coefficient is the average relative loss in the SNR *per loop* over ΔL loops, once the traveling wave has been stored in the ring. In Fig. 2 we plot these coefficients (expressed in %) as a function of D.

To evaluate Eqs. (5) and (6) we have considered a ring of eight oscillators and performed several integrations involving 4, 8, and 16 complete loops to the ring (with τ corresponding respectively to 32 768, 16 384, and 8192 integration steps). We have checked that the losses of SNR in the three cases are consistent with each other and that the relative loss per loop is largely independent of the value of ΔL. The values that are plotted in the figure correspond to averages over these integrations.

As far as the effect of noise is concerned, the system is seen to present three different regimes. The first one is for $D<0.60D_{SR}$. For these reduced levels of noise, information is not stored at all. This is revealed by a very high value of A_{in}. In spite of the fact that the excited oscillator displays a significant SNR, this is not reliably transmitted along the

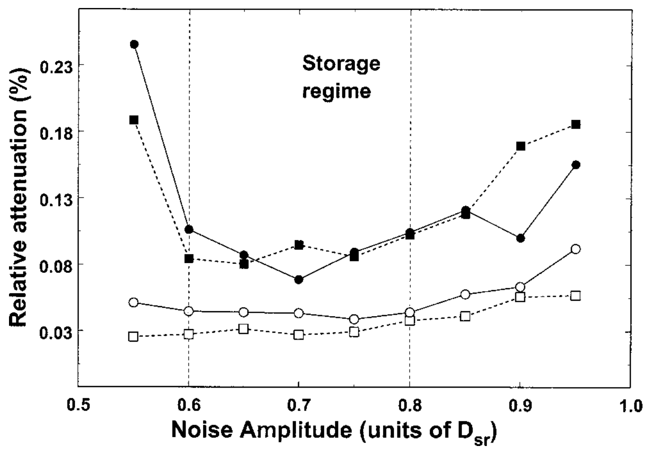

FIG. 2. The coefficients A_{in} (solid symbols) and A_{st} (open symbols) are plotted as functions of D in units of D_{SR}. Circles correspond to $\epsilon'=80$ and squares to $\epsilon'=75$. The ring involves $N=8$ links traversed eight times, and τ equals $16T_{ex}$ (i.e., 16 384 integration time steps). The two vertical dotted lines limit the region of good storage.

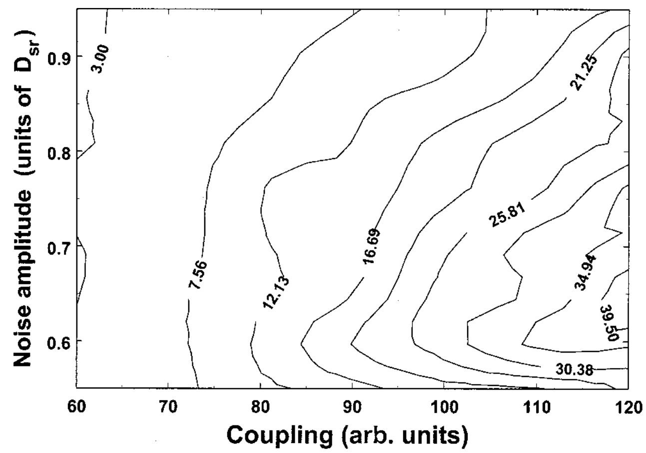

FIG. 3. Contour plot of T_{mem} as a function of D and ϵ'. D is expressed in units of D_{SR}. The values of T_{mem} are given in units of $N\tau$. The other parameters are the same as in Fig. 2.

ring. In this regime A_{st} is not significant. A second situation is found for intermediate values $0.60D_{SR}<D<0.8D_{SR}$. This can be called the "storage regime" because both A_{in} and A_{st} are low and are very similar to each other. This indicates that a sizable signal has been injected into the system and that this is able to live for a long time. The optimal value of D is the same found in Ref. [13]. The fact that $D\sim D_{SR}/\sqrt{2}$ can be understood because there are two sources of noise that are fed into each link of the ring coming from two uncorrelated sources: one that feeds specifically each link and the other that has been fed into the preceding link of the chain and is combined with its periodic oscillation to form its driving force.

For higher levels of noise both coefficients grow steadily, indicating that the system is driven outside the regime of stochastic resonance: neither enhanced periodic oscillations nor transmission and storage are possible anymore.

We choose to define the damping time scale T_{mem} introduced above by assuming that the input SNR undergoes an exponential decay during the next loops, i.e., $R(t)\sim R(L=1)\exp(-t/T_{mem})$ with $t=N\tau L$. We can therefore write

$$\frac{T_{mem}}{N\tau}=\frac{L}{[R(1)-R(L)]_{dB}}. \tag{7}$$

This is justified by realizing that within the storage regime and for moderate values of ϵ', the plots of the R_{dB} vs loop number are very nearly straight lines. T_{mem} is not related separately to A_{st} or A_{in} but rather to an average of both.

In Fig. 3 we show a contour plot of T_{mem} expressed in units of $N\tau$ (or equivalently in turns) as a function of D and ϵ'. The overall pattern of Fig. 3 is similar to that of the correlation length in an open chain of oscillators [13]. They have similar physical significances although they refer to different systems. In the case of an open chain the system is stationary, because the external driving of the chain is never turned off. The present situation corresponds to a time-decaying process, because the external driving is turned off after $N\tau$ units of time and the wave is progressively attenuated from that moment on.

We have determined which values of ϵ' correspond to the driving signal being smaller than the noise perturbation. We find that $\epsilon'>\epsilon$ as expected. Clearly the optimal noise amplitude depends upon ϵ' because this coupling magnifies both the (quasi)periodic signal of the preceding oscillator and the superimposed noise. For very low values of D there is no storage, because the driven oscillator is not within the stochastic resonant regime. For very low ϵ', storage is not satisfactory because the oscillators are nearly uncoupled from each other. Thus as expected, storage is also limited for large levels of noise, independent from the value of ϵ'.

IV. CONCLUSIONS

We have shown the possibility that a temporal, external periodic stimulus persist as a traveling wave in an extended system that is driven by noise. The physical process underlying such short-time storage is that of SR. We have considered a chain of bistable oscillators that operate in that regime. The presence of external noise is essential in sustaining the stored information for an appreciable time once the external stimulus has disappeared. The storage time during which the amplitude of the traveling wave has fallen to a given fraction of its initial value can then be much longer than the transmission delay along the chain or the duration of the stimulus.

The storage time is a tradeoff between the intensity of the directional coupling of the oscillators and the amplitude of the external noise. The fact that all the links of the chain operate in the regime of SR allows for the directional coupling along the chain (as well as the one with the external periodic driving) to be rather weak.

Besides its possible practical applications, this setup is an example of a possible mechanism by which information of the environment can leave reverberant traces in neural loops of the central nervous system. Ever since the suggestion of David Hartley in 1749, it has been repeatedly argued that very short-term memories could well be represented by such excitation patterns. If stimuli persist enough, the short-term reverberant neural loops can stabilize through a Hebbian mechanism, giving rise to permanent synaptic alterations that correspond to long-term memories imprinted in the system.

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